National Institute for Space Research (INPE)

Graduate Program in Space Geophysics (PPGGES)

GES-207-4

Mathematical Methods of Physics I

Course Syllabus and Schedule

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Course Syllabus

GES-207-4 — Mathematical Methods of Physics I (Required Course)
Classes are held on Tuesdays and Thursdays, with 120-minute sessions, from September 22 to December 15, 2026. The course combines lectures, problem solving, student presentations, and two written examinations.

Course Content

Linear Spaces: linear operators, vectors, vector algebra, gradient, divergence, curl and Laplacian; applications in Physics; matrices; tensors; change of vector basis; dual space; eigenvalue problems; matrix diagonalization; Hilbert spaces.

Functions of a Complex Variable: analytic functions, analytic continuation, singularities, residues, the residue theorem, integration of single-valued functions, poles on integration contours, Riemann surfaces, and integration of multivalued functions.

Differential Equations: linear differential equations with constant coefficients; linear differential equations with variable coefficients; Euler differential equations; power-series solutions of linear differential equations; indicial equations; and differential equations of particular importance in Physics.

Special Functions: Legendre polynomials, Bessel functions, and Hermite polynomials.

Integral Transforms: Fourier transforms, Fourier sine and cosine transforms, Laplace transforms, and convolution theorems.

Partial Differential Equations: classification of partial differential equations; Navier–Stokes, wave, diffusion, and Maxwell equations; and the method of characteristics.

Course Organization

The course is organized into five modules. The first two modules develop the linear-algebraic, geometric, and complex-analysis foundations of the course. Modules 3–5 address ordinary differential equations, special functions, integral transforms, and partial differential equations.

Selected preparation problems will be assigned at the end of Thursday classes. Students should prepare *both* problems before the following Tuesday. At the beginning of Tuesday's class, two students will be randomly selected to present the solutions, one problem per student. Each presentation will last approximately five minutes and should emphasize the mathematical reasoning, the principal steps of the solution, and the interpretation of the result.

The standalone lecture notes are intended to serve as the primary study material for the course. They contain the definitions, derivations, demonstrations, examples, physical interpretations, and mathematical details developed in class. Preparation problems are included in the student notes, but their solutions are not provided.

Assessment

The course includes two written examinations:

- **Exam 1 (P1): November 17, 2026 (Tuesday).**
  Coverage: Lectures 1–13 — Linear Spaces and Hilbert Spaces, and Functions of a Complex Variable.
- **Exam 2 (P2 — Final): December 15, 2026 (Tuesday).**
  Coverage: Lectures 14–23 — Ordinary Differential Equations, Special Functions, Integral Transforms, and Partial Differential Equations.

Important schedule note. Exam 1 is scheduled for **November 17, 2026**. Lectures 14–16, which belong to the content assessed in Exam 2, will begin before Exam 1. Exam 1 nevertheless covers only Lectures 1–13.

Bibliography

- ARFKEN, G. B.; WEBER, H. J.; HARRIS, F. E. *Mathematical Methods for Physicists: A Comprehensive Guide*. 7th ed. Academic Press, 2013.
- BYRON JR., F. W.; FULLER, R. W. *Mathematics of Classical and Quantum Physics*. Reading, MA: Addison-Wesley, 1970.
- MACHADO, K. D. *Equações diferenciais aplicadas à Física*. Editora UEPG, 1990.
- MEDRANO-B., R. A. *Transformadas Integrais*. São José dos Campos: INPE, April 1989. (INPE-4807-MD/40).
- MEDRANO-B., R. A. *Equações diferenciais ordinárias*. São José dos Campos: INPE, May 1980. (INPE-1753-RPE/144).
- MEDRANO-B., R. A. *Campos vetoriais, espaços lineares e tensores na física*. São José dos Campos: Transtec, 1994.
- MEDRANO-B., R. A. *Funções de variáveis complexas*. São José dos Campos: INPE, May 1979. (INPE-1494-RPE/038).
- MEDRANO-B., R. A. *Funções especiais*. São José dos Campos: INPE, February 1985. (INPE-3479-RPE/472).
- OCKENDON, J. R.; HOWISON, S. D.; LACEY, A. A.; MOVCHAN, A. B. *Applied Partial Differential Equations*. Oxford University Press, 2003.

Course Schedule

The schedule below presents the planned sequence of topics. Dates may be adjusted if necessary according to the academic calendar. Any changes will be communicated to the students in advance.

Lecture	Date	Module	Topic
01	Sep. 22 (Tue)	1. Linear Spaces and Hilbert Spaces	Matrices and Vector Spaces
02	Sep. 24 (Thu)		Inner Product, Norm, and Orthogonality
03	Sep. 29 (Tue)		Gram–Schmidt, Orthonormal Bases, and Projections
04	Oct. 1 (Thu)		Eigenvalues, Diagonalization, and Matrix Functions
05	Oct. 6 (Tue)		Hermitian Operators and the Spectral Theorem
06	Oct. 8 (Thu)		Hilbert Spaces and the Space L^2
07	Oct. 13 (Tue)		Vector Calculus
08	Oct. 15 (Thu)		Tensor Algebra
09	Oct. 20 (Tue)		Tensors and Applications
10	Oct. 22 (Thu)	2. Complex Variable	Analytic Functions and the Cauchy–Riemann Conditions
11	Oct. 27 (Tue)		Complex Integration and Cauchy's Theorem
12	Oct. 29 (Thu)		Taylor and Laurent Series, Singularities, and Residues
13	Nov. 3 (Tue)		Poles on the Contour and Riemann Surfaces
14	Nov. 5 (Thu)	3. Ordinary Differential Equations	First-Order ODEs: Integrating Factor, Bernoulli, and Riccati Equations
15	Nov. 10 (Tue)		Second-Order Linear ODEs with Constant Coefficients
16	Nov. 12 (Thu)		Variable-Coefficient ODEs and the Euler Equation
P1	**Nov. 17 (Tue)**	**Assessment**	**Exam 1 — Lectures 1–13**
17	Nov. 19 (Thu)	3. Ordinary Differential Equations	Power Series, Frobenius Method, and the Indicial Equation
18	Nov. 24 (Tue)	4. Special Functions	Legendre Polynomials
19	Nov. 26 (Thu)		Bessel Functions
20	Dec. 1 (Tue)		Hermite Polynomials
21	Dec. 3 (Thu)	5. Integral Transforms and PDEs	Fourier, Sine, and Cosine Transforms
22	Dec. 8 (Tue)		Laplace Transform and Convolution

Lecture	Date	Module	Topic
23	Dec. 10 (Thu)		Partial Differential Equations, Method of Characteristics, and General Review
P2	**Dec. 15 (Tue)**	**Assessment**	**Exam 2 (Final) — Lectures 14–23**